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DERIVE A NEW RULE FOR FINDING THE VALUES OF NUMERICALLY DEFINED ONE SIDED INTEGRALS

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Abstract:
This study's primary objective is to devise a new rule to determine the values of numerically defined fundamentals of continuous functions. Although they are continuous, they are defective in the derivative or defective at one or more points in the integration region.[1] Error formulas were found for them as we deduced this method, where the harmonic mean was taken. For the trapezoid and Simpson methods, then comparing the results with numerical integration methods and adopting the percentage of absolute error, [3]it was found that the new method is better than the previous methods. We concluded that we can rely on this method in calculating definite integrals, as it gave high accuracy in the results.

Keywords: Harmonic Mean, trapezoidal Rule, Simpson's 1/3- rule.

1. Introduction.

The following is a possible formulation of the generic numerical integration problem:

With a collection of data points [10].

$((x_{0,}p_0),(x_{1,}p_1),\ldots..(x_n,p_n)$ of an operation

$h = f(x),$ as well as $f(x)$ Is reguired To calculate the worth of the integral that is definite as:

$$I = \int_v^q p(x)dx \qquad (1)$$

As When numerical differentiation is applied, one replaces $f(x)$ by a polynomial interpolator

$\emptyset(x)$ and gets,[7] on integration a rough estimate obtained depending upon that type of the interpolation One can obtain formulas based on the kind of interpolation formula that was applied. Within this section a universal numerical integration formula will be derived with the use of Newtons formula for forward difference. [2]

Allow the time gap $[v, q]$be separated in to n equal subintervals in which

$$v = x_0 < x_1 < x_2 < x_3 < \cdots < x_n = q. \text{ Clearly,} x_n = x_0 + nh.$$

The integral follows.

$$I = \int_{x_0}^{x_n} p(x)dx \qquad (2)$$

estimating $f(x)$ using Newton's forward difference formula, we arrive at

$$I = \int_{x_0}^{x_1} h[p_0 + u\Delta p_0 + \frac{l(l-1)}{2!}\Delta^2 p_0 + \frac{l(l-1)(l-2)}{3!} + \cdots +]dx \qquad (3)$$

Since $x = x_0 + mh$, $dx = hdm$ and hence the above integral becomes:

$$I = h\int_0^n [p_0 + u\Delta p_0 + \frac{l(l-1)}{2!}\Delta^2 p_0 + \frac{l(l-1)(l-2)}{3!}\Delta^3 p_0 + \cdots +]\, dl \qquad (4)$$

Which gives on simplification

$$\int_{x_0}^{x_n} p(x)dx = nh[p_0 + \frac{n}{2}\Delta p_0 + \frac{n(2n-3)}{12}\Delta^2 p_0 + \frac{n(n-2)^2}{24} + \Delta^3 p_0 + \cdots +] \qquad (5)$$

based on this basic principle. Various integration formulas can be obtained by substituting

$n = 1,2,3, \ldots.,ets.$ As we can Here, obtain many of these equations. , such As the 1/3-rule for Simpson and the trapezoidal which are determined to provide adequate accuracy. to be used practically real life difficulties [11].

2. Rule of Trapezoids:

Positioning $n = 1,$ in the overall formula (5), Every difference that is greater than the first will be equal to zero, giving us [8],

$$\int_{x_0}^{x_1} p dx = h[p_0 + \frac{1}{2}\Delta p_0] = h[p_0 + \frac{1}{2}(p_1 - p_0) = \frac{h}{2}(p_0 + p_1) \qquad (6)$$

For the upcoming time frame $[x_1, x_2]$, the formula Will deduce similarly as:

$$\int_{x_1}^{x_2} p dx = \frac{h}{2}(p_1 + p_2) \qquad (7)$$

And thus, during the final intermission $[x_{n-1}, x_n]$, We've got

$$\int_{x_{n-1}}^{x_n} p dx = \frac{h}{2}(p_{n-1} + p_n) \qquad (8)$$

By combining each of these phrases, we can acquire the following governing :

$$\int_{x_0}^{x_n} p dx = \frac{h}{2}[p_0 + 2(p_1 + p_2 + p_3 + \cdots + p_{n-1}) + p_n] \qquad (9)$$

This goes under the name of the trapezoidal rule. In terms of geometry, this rule means that the curve $p = p(x)$ is replaced by n stratight lines joining the points $(x_0, p_0)(x_1, p_1)$;

Additionally $(x_1, p_1) and (x_2, p_2), \ldots, (x_{n-1}, p_{n-1}) and\ (x_n, p_n)$. The region is contained within the curve. $p = p(x)$ the ordinates $x = x_0\ and\ x = x_n$, along with The

x-axis is is then roughly equal to the total of the areas within the n trapeziuums acquired.

The following method can be used to determine the trapezoidal formula error: Allow

$p = p(x)$ be a consistent, okay -defined, They have constant derivatives in $[x_0, x_1]$. Expanding g within a Taylor's series centered $x = x_0$, We'll get

$$\int_{x_0}^{x_1} p dx = \int_{x_0}^{x_1} [p_0 + (x - x_0)p_0' + \frac{(x-x0)^2}{2} p_0'' + \cdots] dx$$

$$= hp_0 + \frac{h^2}{2} p_0' + \frac{h^3}{6} p_0'' + \cdots] \qquad (10)$$

Similarly

$$\frac{h}{2}(p_0 + p_1) = \frac{h}{2}[p_0 + p_0 + hp_0' + \frac{h^2}{2} p_0'' + \frac{h^3}{6} p_0''' + \cdots]$$

$$= hp_0 + \frac{h^2}{2} p_0' + \frac{h^3}{4} p_0'' + \cdots \ ] \qquad (11)$$

From Eqs.(10) and (11) we obtain

$$\int_{x_0}^{x_1} p dx - \frac{h}{2}(p_0 + p_1) = -\frac{1}{12} h^3 p_0'' + \cdots] \qquad (12)$$

That is the errors during the pause $[x_0, x_1]$. Continuing along the same path, we derive the errors in the remaining subintervals, for example. $[x_1, x_2], [x_2, x_3], \ldots . and\ [x_{n-1}, x_n]$. Consequently, have

$$E = -\frac{1}{12} h^3 n p''(\bar{x}) = -\frac{q-v}{12} h^2 p''(\bar{x}), \qquad (13)$$

Since

$$h = \frac{q-v}{n}$$

3. Simpson's 1/3- rule

This rule is obtained by $n = 2\ in\ eq.(\ 5), i.e.$ through substituting n/2 arcs of second-degree either parabolas or polynomials.Next, we have [14]:

$$\int_{x_0}^{x_2} pd(x) = 2h[p_0 + \Delta\, p_0 + \frac{1}{6}\Delta^2 p_0]$$

$$= \frac{h}{3}(p_0 + 4p_1 + p_2). \qquad (14)$$

In a similar vein,

$$\int_{x_2}^{x_4} pd(x) = \frac{h}{3}(p_2 + 4p_3 + p_4) \qquad (15)$$

And lastly

$$\int_{x_{n-2}}^{x_n} pd(x) = \frac{h}{3}(p_{n-2} + 4p_{n-1} + p_n) \qquad (16)$$

By In summary, we will receive

:

$$\int_{x_0}^{x_n} pd(x) = \frac{h}{3}(p_0 + 4(p_0 + p_3 + p_5 + \cdots + p_{n-1}) + 2(p_2 + p_4 + p_6 + \cdots + p_{n-2)} + p_n) \qquad (17)$$

This is referred to as Simpson's rule, Simpson's 1/3-rule, or just Simpson . [13] Note that the total range must be divided into an even number of subintervals of width h in order to comply with this condition.

4. The Harmonic Mean

The data value's inverse of the average of its inverses. In light of every observation

, By assigning more weight to the little values and less weight to the large values, the harmonic mean accurately balances the data. Overall,[18]

$$HM = \left(\frac{\sum_{i-1}^{n} K_i}{n}\right)^{-1} \qquad (18)$$

We derive the new rule from the trapezoid and Simpson rules by finding the harmonic mean of the functions, not the points of the two rules

$$(19) N = \int_a^b f(x)dx = h\left[\frac{2}{\frac{1}{\frac{fa+f_b}{2}} + \frac{1}{\frac{fa+f_b}{3}}} + \frac{2}{\frac{1}{\frac{4}{3}}+1} f_i^{n-1} + \frac{2}{\frac{1}{\frac{2}{3}}+1} f_i^{n-2}\right]$$

$$N = \int_a^b f(x)dx = h\left[\frac{2}{\frac{2}{fa+f_b} + \frac{3}{fa+f_b}} + \frac{2}{\frac{3}{4}+1} f_i^{n-1} + \frac{2}{\frac{3}{2}+1} f_i^{n-2}\right] \qquad (20)$$

$$N = \int_a^b f(x)dx = h\left[\frac{2}{\frac{5}{fa+f_b}} + \frac{2}{\frac{7}{4}} f_i^{n-1} + \frac{2}{\frac{5}{2}} f_i^{n-2}\right] \qquad (21)$$

$$N = \int_a^b f(x)dx = h\left[2 * \frac{f_a+f_b}{5} + 2 * \frac{4}{7} f_i^{n-1} + 2 * \frac{2}{5} f_i^{n-2}\right] \qquad (22)$$

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$$N = \int_a^b f(x)dx \; = h\left[\frac{2(f_a+f_b)}{5} + \frac{8}{7} f_i^{n-1} + \frac{4}{5} f_i^{n-2}\right] \qquad (23)$$

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$$N = \int_a^b f(x)dx = h\left(\frac{2(f_a+f_b)}{5} + \frac{8}{7}\sum_{i=1,3,5}^{n-1} f i_{i=1,3,5....} + \frac{4}{5}\sum_{i=2,4,6}^{n-2} f i_{i=2,4,6....}\right)$$

(24)

Error = | precise value - approximate amount |

5.Example

Calculate $\int_0^{\pi} sin(x)\, dx$ and compare the answers.

Exact amount of $\int_0^{\pi} sin(x)\, dx = 0.08610697004$, $h = \frac{\pi}{6}$

x	0	π/6	π /3	π /2	2π /3	5π /6	π
y	0.0	0.5	0.8660	1.0	0.8660	0.5	0.0

result from the suggested method

$$N = \frac{\pi}{6}[\,0.6 + 1.807085714 + 0.64]$$

N = 1.922295665

Error=| 0.08610697004 - 1.922295665 |= 1.836188695

Table (1) Examples of the proposed method in comparison with theSimpson's 1/3- method and the trapezoidal method

Operation	True Value	The trapezoid's rule	mistake percentage	Simpson's 1/3-Rule	mistake percentage	Suggested Method	mistake percentage
$\int_1^2 \frac{\sin(x)}{x} dx$	0.01745122505	0.6052853295	0.5878341044	0.6593333333	0.641882108	0.5974967252	0.5800455001
$\int_1^5 log(x)\, dx$	1.757672094	4.0307	2.273027906	4.0467	2.289027906	3.923765715	2.166093621
$\int_0^1 \frac{1}{2+\cos(x)} dx$	0.3333389745	0.2972088423	0.0361301322	0.3527953333	0.1994563588	0.3427675586	0.0094285841
$\int_0^{\pi} sin(x)\, dx$	0.08610697004	1.9540	1.86789303	2.0008	1.91469303	1.922295665	1.836188695
$\int_1^2 \frac{1}{1+x} dx$	0.4054651081	0.4581125	0.0526473919	0.4747083333	0.0692432252	0.4535585714	0.0480934633
$\int_0^{\pi/2} \sqrt{\sin(x)} dx$	0.1733866207	1.146956208	0.9735695873	1.178228145	1.004841524	1.130268741	0.9568821203

6. Conclusions

We conclude from the above results that when calculating single integrals with continuous integrals using the trapezoidal base and the new base, we obtained accurate figures to compared to several decimal places [17] With that Exact virtues . For integrals and with the use of a small how many subintervals exist without using that process outside of modification to them, we obtained tow decimal places in the first and third examples, and the number of subintervals to obtain was On the tow ranks in the new rule,[4] their number is much less than in the trapezium rule, and on three decimal places in the second example. Also, the number of partial periods in the new rule was less,[6] and this indicates the superiority of the method.

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