

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

# TEACHING SIMILAR TRIANGLES, HOMOTHETY, QUADRILATERALS, AND CIRCLES IN SECONDARY SCHOOL GEOMETRY: A LITERATURE REVIEW

Sattorov Ilxomjon Toshpo'latovich

Namangan State Technical University Academic Lyceum

### Abstract

Similar triangles, homothety, quadrilaterals, and circles constitute a core sequence of topics in secondary school plane geometry, yet each presents distinct pedagogical challenges related to students' developmental readiness for formal geometric reasoning. This paper reviews recent literature on teaching methodologies for this topic cluster, drawing primarily on studies published between 2023 and 2025. The review addresses three interconnected themes: the van Hiele model of geometric thinking as a diagnostic and instructional framework, particularly as applied to student misconceptions about quadrilaterals; task-design approaches to introducing homothety and similarity transformations, informed by the Mathematical Working Space framework; and the role of dynamic geometry software, particularly GeoGebra, in supporting conceptual understanding across this topic sequence. Across this literature, a recurring finding is that a substantial proportion of secondary students remain at pre-formal (visualization or early analysis) levels of geometric thinking when engaging with quadrilateral classification and transformation concepts, and that textbook presentation of these topics, particularly homothety, often contains structural inconsistencies that compound these difficulties. The review concludes that effective instruction in this domain requires explicit attention to students' van Hiele level, carefully sequenced task design, and integration of dynamic geometry tools rather than reliance on static diagrams alone.

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

**Keywords:** Similar triangles; homothety; quadrilaterals; circles; van Hiele model; geometry education; GeoGebra.

### Introduction

The sequence of topics comprising similar triangles, homothety (dilation), quadrilaterals, and circles forms a foundational component of secondary school plane geometry curricula internationally, bridging students' earlier experience with basic shape recognition and the more formal deductive reasoning required in later geometric proof. Despite the apparent conceptual continuity of this sequence, from proportional reasoning in similar triangles, through the transformation geometry of homothety, to the classification hierarchies of quadrilaterals and the metric properties of circles, research indicates that students frequently experience persistent difficulties across each of these subtopics, difficulties that are often rooted in mismatches between the level of formal reasoning assumed by curricula and textbooks and students' actual developmental readiness for such reasoning.

This paper reviews recent literature, published predominantly between 2023 and 2025, on teaching methodologies and pedagogical approaches for this topic cluster. The review is organized around three interconnected themes identified as recurring foci in this literature: the van Hiele model of geometric thinking as both a diagnostic tool for identifying student misconceptions and a framework for sequencing instruction; task-design research specifically addressing the introduction of homothety and similarity transformations; and the growing role of dynamic geometry software in supporting students' transition toward more formal geometric understanding across this topic sequence.

### The van Hiele Model and Student Misconceptions About Quadrilaterals

The van Hiele model of geometric thinking, originally developed in the 1950s and remaining highly influential in contemporary geometry education research,

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

describes student understanding as progressing through a sequence of qualitatively distinct levels: visualization, in which shapes are recognized by overall appearance rather than by properties; analysis, in which students can describe and classify shapes according to their properties without yet grasping logical relationships between those properties; and subsequent levels of informal and formal deduction.

A case study examining seventh-grade students' van Hiele levels specifically with respect to quadrilaterals found persistent misconceptions concerning the concepts of diagonal and height, and regarding special quadrilaterals more broadly; notably, students operating at the analysis level were found to have difficulty recognizing a square positioned with a vertex pointing downward as a square, whereas students at the visualization level, somewhat counterintuitively, recognized the shape correctly, suggesting that formal property-based reasoning can, in certain configurations, interfere with rather than support shape recognition (Satır & Kurtuluş, 2024).

A related case study conducted in a Papuan junior high school found that fully 50% of students operated at van Hiele Level 0 (visualization) with respect to quadrilaterals, with a further 36% transitioning between Level 0 and Level 1 and only 14% having reached Level 1 (analysis), and that students frequently relied on the visual appearance of shapes rather than their formal geometric attributes, a pattern of difficulty that persisted despite direct instruction. This body of research collectively suggests that quadrilateral instruction predicated on formal classification and property-based reasoning may substantially outpace the actual van Hiele readiness of a considerable proportion of secondary students, underscoring the diagnostic value of van Hiele-based assessment prior to and during instruction on this topic.

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

### Task Design for Teaching Homothety and Similarity

Homothety, as a specific geometric transformation underlying the broader concept of similarity, has received focused attention in recent task-design research. A study examining the design and implementation of a task sequence for teaching homothety within the Chilean educational context, framed using the Mathematical Working Space (MWS) construct, found that reference MWS frameworks require careful adaptation for classroom use, and that textbooks in the Chilean context often present homothety with structural inconsistencies that can further challenge student understanding beyond the inherent conceptual difficulty of the topic itself (Hernández & Caviedes, 2024). The study emphasized the redesign of tasks by a teacher-in-training during supervised practicum experience, highlighting the practical importance of preparing prospective mathematics teachers with task-design and adaptation skills specific to this topic, rather than relying solely on textbook sequences as originally presented.

Complementing this task-design focus, a comparative analysis of congruent and similar triangle units across mathematics textbooks from China and the United States found that textbooks function as a critical bridge between intended national curriculum standards and actual classroom-implemented instruction, and that the two educational systems examined exhibited distinctly different pedagogical approaches to presenting this topic area, reflecting differing underlying assumptions about how students should be led toward formal proof-based reasoning about triangle similarity and congruence (Zhou, Lo, & Liu, 2024). Together, these studies suggest that textbook design, rather than being pedagogically neutral, substantially shapes the instructional pathway by which students encounter similarity and homothety concepts, and that comparative and design-focused research in this area remains a productive avenue for identifying more effective sequencing.

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

### Dynamic Geometry Software and Conceptual Understanding

The integration of dynamic geometry software, particularly GeoGebra, has emerged as a prominent theme in recent literature addressing instruction across the quadrilateral-circle topic range. A study examining GeoGebra-assisted instructional methods found measurable effects on students' conceptual understanding of geometry, attributed to the software's capacity to allow students to manipulate figures dynamically, for instance by dragging vertices while properties and measures update in real time, rather than relying on static diagrams that present only a single configuration of a given shape (Gurmu, Tuge, & Hunde, 2024). This affordance is particularly relevant to the quadrilateral and circle topics reviewed here, since many of the classification difficulties documented in van Hiele-based studies stem precisely from students' overreliance on the visual appearance of a single static example rather than an understanding of the underlying properties that remain invariant under transformation.

A related design study examined STEM-integrated approaches to quadrilateral instruction using a lesson study methodology, developing learning designs intended to connect quadrilateral geometry to applied problem contexts rather than presenting classification and property-based content in isolation (Nursyahidah, Albab, & Mulyaningrum, 2023). This line of research suggests a broader pedagogical trend in the reviewed literature toward embedding quadrilateral and circle geometry within applied, dynamically manipulable, or STEM-integrated contexts, as a means of supporting students' progression beyond visualization-level reasoning toward the analysis and informal deduction levels required for subsequent formal geometric proof.

### Discussion

Taken together, the literature reviewed here indicates that instructional difficulty in the similar triangles, homothety, quadrilaterals, and circles topic sequence is not primarily attributable to the inherent conceptual difficulty of the mathematical

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

content alone, but substantially to a mismatch between the level of formal, property-based, and transformation-based reasoning assumed by curricula and textbooks and the actual van Hiele-level readiness of a considerable proportion of secondary students, a mismatch that recent task-design and textbook-comparison research suggests is compounded in some cases by structural inconsistencies within textbook presentation itself (Hernández & Caviedes, 2024).

The consistent finding across van Hiele-based studies that large proportions of students remain at pre-analysis levels when engaging with quadrilateral content carries direct implications for how homothety and similarity, which presuppose a degree of comfort with property-based and proportional reasoning, should be sequenced relative to quadrilateral instruction; the reviewed literature suggests that diagnostic van Hiele assessment prior to introducing transformation-based content may allow instructors to better calibrate the level of formal reasoning assumed in task design. The prominence of dynamic geometry software across this literature further suggests that the visualization-to-analysis transition documented in van Hiele research may be more effectively supported through tools that allow direct manipulation of geometric invariants than through static textbook diagrams, though the reviewed studies stop short of establishing controlled comparative evidence isolating this specific mechanism.

Limitations of the reviewed literature include the geographic and grade-level concentration of several key studies, for instance the seventh-grade Turkish and Papuan contexts for van Hiele-based quadrilateral research and the Chilean context for homothety task design, which may limit generalizability of specific findings to other curricular and cultural contexts; and the predominantly qualitative, case-study-based methodology of much of this research, which, while well suited to documenting the nature of student misconceptions, provides comparatively limited evidence regarding the relative effectiveness of specific instructional interventions at scale.

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

### Conclusion

This review of recent literature on teaching similar triangles, homothety, quadrilaterals, and circles indicates that student difficulty across this topic sequence is substantially shaped by the gap between curricular assumptions of formal geometric reasoning and students' actual van Hiele-level readiness, a gap that recent research suggests can be addressed through careful task design informed by frameworks such as the Mathematical Working Space, critical attention to textbook presentation consistency, and integration of dynamic geometry software to support the transition from visualization-based to property-based and transformation-based reasoning. Future research would benefit from more extensive quantitative and cross-cultural comparative work to establish the relative effectiveness of specific instructional sequences and technological interventions across this topic cluster.

### Funding

This research received no external funding.

### Conflict of Interest

The authors declare no conflict of interest.

### References

1. Gurmu, F., Tuge, C., Hunde, A.B., 2024. Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education* 11(1), 2379745.
2. Hernández, C., Caviedes, S., 2024. Design and implementation of a task sequence for teaching homothety. *Eurasia Journal of Mathematics, Science and Technology Education* 20(9).

## Eureka Journal of Education & Learning Technologies (EJELT)

ISSN 2760-4918 (Online)

Volume 2, Issue 9, September 2026



This article/work is licensed under CC by 4.0 Attribution

<https://eurekaoa.com/index.php/2>

3. Nursyahidah, F., Albab, I.U., Mulyaningrum, E.R., 2023. Learning design of quadrilateral STEM-based through lesson study. Eurasia Journal of Mathematics Science and Technology Education 19(11), em2352.
4. Satır, S., Kurtuluş, A., 2024. Examination of seventh grade students' van Hiele geometric thinking levels and their mistakes on 'quadrilaterals'. Osmangazi Journal of Educational Research 10(2), 23-51.
5. Zhou, L., Lo, J.J., Liu, J., 2024. A comparative analysis of congruent triangle units in textbooks from China and the United States. Journal of Curriculum Studies Research.