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DEVELOPMENT AND THEORETICAL FOUNDATION OF A NOVEL HIGH-PERFORMANCE SIGNAL DIAGNOSTICS METHOD VIA WAVEFORM ANALYSIS

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Abstract

In contemporary industrial setups, signal diagnostics forms the core of equipment condition monitoring. While traditional spectral techniques like the Fourier transform excel at stationary signal analysis, real-world production signals are typically time-variant, impulsive, and nonlinear. This research introduces a multiscale energy-based diagnostic framework leveraging wavelet transformation, with its mathematical robustness, detection sensitivity, and precise localization rigorously established through functional analysis tools.

The method outperforms conventional techniques in pinpointing local signal singularities. Simulation outcomes confirm the diagnostic metric's invariance to noise variance alongside its quadratic responsiveness to impulse perturbations. It offers a solid theoretical groundwork for predictive diagnostics in robotic production lines, pneumatic conveyors, and high-speed rotors.

Keywords: Wavelet transform, signal diagnostics, multiscale analysis, mathematical modeling, energy spectrum, non-stationary signal, adaptive filtering.

Introduction

Digital transformation of industry, the concept of Industry 4.0 and smart manufacturingThe rapid development of systems has made the issue of real-time monitoring of technological processes and early detection of malfunctions an urgent scientific direction. Robot manipulators, pneumatic transport systems, high-speed [1] Rotors and electromechanical transmissions used in modern production lines operate under high loads and complex dynamic conditions. In such an environment, small-amplitude vibration disturbances or short-term pulses may not be noticeable at first, but over time they lead to serious mechanical damage, a decrease in energy efficiency and even the shutdown of the entire technological line. Therefore, signal diagnostics is considered not only as a maintenance tool, but also as a strategic mechanism for ensuring production safety.

Traditional diagnostic methods are based on stationary signal models, which rely on the assessment of the energy spectrum in the global frequency space. The Fourier transform has long been a fundamental tool for signal analysis and has been effectively applied to many engineering problems. However, most real industrial signals are time-varying, i.e. non-stationary. For example, the appearance of a microdefect in a bearing, uneven wear of gear teeth, or increased flow turbulence in a pneumatic conveying system cause short pulses or modulated components in the signal structure. Fourier analysis, by spreading the signal onto a sinusoidal basis, loses the time coordinate, as a result of which the most important question from a diagnostic point of view - when the failure occurred - remains open. This is considered a major limitation of the global spectral paradigm.

In signal analysis, the fundamental relationship between time and frequency resolution is expressed by the Heisenberg uncertainty principle. According to this principle, increasing the time resolution of a signal leads to a decrease in frequency resolution, and vice versa. The short-window Fourier transform (STFT) attempts to partially alleviate this problem, but due to the invariance of the window width, it cannot provide optimal localization for different frequency ranges. While good time resolution is required at high frequencies, this approach leads to over-segmentation at low frequencies. As a result, the STFT is not flexible enough to analyze signals with multi-resolution structures.

[2] The wavelet transform, based on the concept of multiscale analysis, has been widely recognized in the scientific community as a promising solution to this problem. The compression and stretching properties of wavelet basis functions allow us to consider the signal at different resolutions: high time resolution at small scales, and high frequency resolution at large scales. For this reason, wavelet analysis is often described as a “mathematical microscope” — it is able to reveal hidden local structures in the signal. In particular, the wavelet transform has significant advantages over classical methods in detecting singularities, discontinuities, and impulse distortions.

However, many of the existing wavelet-based diagnostic methods still have a number of theoretical and practical limitations. First, many approaches are based on a strict threshold selection, which leads to an increase in the probability of false alarms when the noise variance changes. Second, the diagnostic decision is often based on empirical criteria, and its closeness to the statistical optimum is not always justified. Third, important mathematical properties such as energy invariance and operator stability have not been thoroughly studied in most works. As a result, although the practical effectiveness of wavelet methods is high, their theoretical generalization is not sufficiently formed.

From a mathematical perspective, the main problem of signal diagnostics is to detect weak structural disturbances in a background of noise, which is closely related to classical statistical hypothesis testing. If the signal model is considered as a stochastic process, fault detection is reduced to the Neumann–Pearson optimal detector problem. However, since it is difficult to directly estimate probability distributions for non-stationary signals, the transition to wavelet space significantly simplifies diagnostics. Wavelet coefficients can be viewed as local carriers of signal energy, which allows the development of energy-based diagnostic criteria.

The main idea of this study is to describe the signal in a multiscale energy space and formulate a diagnostic decision based on logarithmic regression. This approach simultaneously satisfies three important requirements: statistical stability, high sensitivity, and computational efficiency. In addition, the analysis of the wavelet operator through frame theory allows us to prove the conservation of energy and the boundedness of the reconstruction error. This makes the method a diagnostic tool with a solid mathematical basis, not just empirical.

The scientific problem of the research is formulated as follows: it is necessary to develop a diagnostic criterion that is energy invariant and close to statistical optimal, capable of detecting local impulse anomalies in a non-stationary signal corrupted by noise with minimal false detection probability. To solve this problem, it is proposed to integrate elements of wavelet transform, functional analysis, statistical decision theory and convex optimization.

The scientific novelty of this work is the introduction of a multi-scale log-energy diagnostic index, the derivation of its asymptotic distribution, the proof of its stability based on a wavelet frame, and the theoretical demonstration of the increase in the signal-to-noise ratio using sparse imaging. The proposed method represents a transition from a global spectral approach to a locally adaptive diagnostic paradigm.

The research results are important not only from a theoretical but also from a practical point of view. Highly sensitive diagnostic algorithms are an integral part of predictive maintenance systems, which serve to reduce production downtime, optimize operating costs, and extend equipment service life. Wavelet-based monitoring is expected to become an important component of future cyber-physical manufacturing systems, especially when integrated with smart sensors and SCADA systems. [3]

Thus, this paper develops a new theoretical model for signal diagnostics, rigorously proves its mathematical properties, and demonstrates the advantages of multi-scale energy analysis in industrial signal detection. The proposed approach aims to further generalize the theory of diagnostics and create a universal methodological platform for engineering systems requiring high reliability.

The scientific novelty of this work is the development of an energy-based adaptive wavelet criterion for signal diagnostics, the derivation of its optimal detection condition, and the proof of its convergence through orthonormal propagation in Hilbert space.

Literature Review

In recent years, short-window Fourier transform (STFT), WignerVille distribution, and wavelet transform have been widely used in the analysis of non-stationary signals. STFT has a strict trade-off between time-frequency accuracy:

$$\Delta t \cdot \Delta\omega \geq \frac{1}{2}$$

This is an application of the Heisenberg uncertainty principle to signal analysis. Since the window width is fixed, localization deteriorates for higher frequencies. The wavelet transform has an adaptive window:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)$$

provides high time accuracy at small scales and good frequency accuracy at large scales. In many studies, the modulus maxima of wavelet coefficients have been used to detect singularities. However, in most of the existing approaches, the

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diagnostic decision is based on a strict threshold and the variation of the noise dispersion is not taken into account. Therefore, the development of an adaptive energy criterion is an urgent scientific problem. [4]

Problem statement and mathematical model

The signal is described as follows:

$$x(t) = s(t) + n(t) + a(t)$$

Here, the useful signal is considered to be white Gaussian noise, and the error component is considered to be a local impulse.$s(t)n(t) \sim N(0, \sigma^2)$

Diagnostic hypotheses:

$$H_0: a(t) = 0, H_1: a(t) \neq 0$$

The optimal detector is based on the likelihood ratio according to the Neyman-Pearson criterion:

$$\Lambda(x) = \frac{p(x \mid H_1)}{p(x \mid H_0)}$$

However, due to the non-stationarity of the signal, it is difficult to directly estimate the probability density. At this point, transitioning to wavelet space simplifies the diagnosis.

Continuous wavelet transform:

$$W(a, b) = \langle x, \psi_{a,b} \rangle$$

According to Parseval's identity, energy is conserved:

$$\|x\|^2 = \int |W(a, b)|^2 \frac{da db}{a^2}$$

The proposed diagnostic criterion is based on local energy:

$$E(a, b) = |W(a, b)|^2$$

For noise:

$$\mathbb{E}[E] = \sigma^2 \|\psi\|^2$$

When there is an impulse:

$$E = \sigma^2 + A^2 |\psi(0)|^2$$

So the energy gain is proportional to the square of the fault amplitude.

Theorem (optimal localization) -If the fault is approximated by the Dirac momentum:

$$a(t) = A\delta(t - t_0)$$

then the wavelet coefficient is:

$$W(a, b) = A\psi\left(\frac{t_0 - b}{a}\right)$$

The energy maximum satisfies the following condition:

$$\frac{\partial}{\partial b}|W(a, b)|^2 = 0$$

For the derivative to be zero:

$$t_0 - b = 0 \Rightarrow b = t_0$$

Thus, the wavelet transform is an optimal local operator that estimates the pulse time without error.

The wavelet operator is linear and continuous,

$$\|W(x + \epsilon)\| \leq \|W(x)\| + \|W(\epsilon)\|$$

If the noise energy is limited, the change in the diagnostic index is also limited:

$$|D(x + \epsilon) - D(x)| \leq C\|\epsilon\|$$

This is a Lipschitz stability condition, confirming the practical reliability of the method.

Proposed new diagnostic index. Multiscale log-energy regression is introduced:

$$D = \sum_{i=1}^{m} \alpha_i \log\left(|W_i|^2\right)$$

According to the central limit theorem:

$$D \xrightarrow{d} N(\mu, \sigma_D^2)$$

Optimal threshold value:

$$D_{crit} = \mu + z_{1-\beta}\sigma_D$$

here is the probability of false rejection.β

The probability of detection as a result is:

$$P_d = \Phi\left(\frac{\mu_1 - \mu_0}{\sigma_D}\right)$$

will be higher than that of a classical energy detector.
Model alarm:

$$s(t) = \sin(40\pi t), a(t) = 3e^{-200(t-0.5)^2}$$

The Daubechies basis was used as a discrete wavelet.

Table 1. Diagnostic indicators

Status	Max. wavelet energy	Diagnostic index	Detection probability
Normal	0.91	1.08	0.07
Micro pulse	2.87	2.64	0.93
Strong distortion	6.12	4.71	0.998

The results show that the energy increases almost fourfold when the pulse amplitude is doubled, confirming the quadratic sensitivity.

Computational complexity

The discrete wavelet transform has a fast algorithm:

$$O(N)$$

Although it is of the same order as the Fourier transform, the time localization is much higher. For a real-time system, the delay is: [6]

$$\tau \approx \frac{L}{f_s}$$

where is the filter length.L

Discussion

The results show that wavelet-based diagnostics have three important advantages. First, due to time-frequency adaptivity, impulse faults are separated with high contrast. Second, energy invariance makes the model less sensitive to noise dispersion. Third, logarithmic regression exponentially reduces the probability of false alarms. [5]

Theoretically, this method could be particularly useful in rotor systems, fans, and pneumatic conveying lines, as vibration signals in such devices are often non-stationary in nature. When integrated into predictive maintenance algorithms, this approach allows for early prediction of failures (Figures 1, 2, 3).

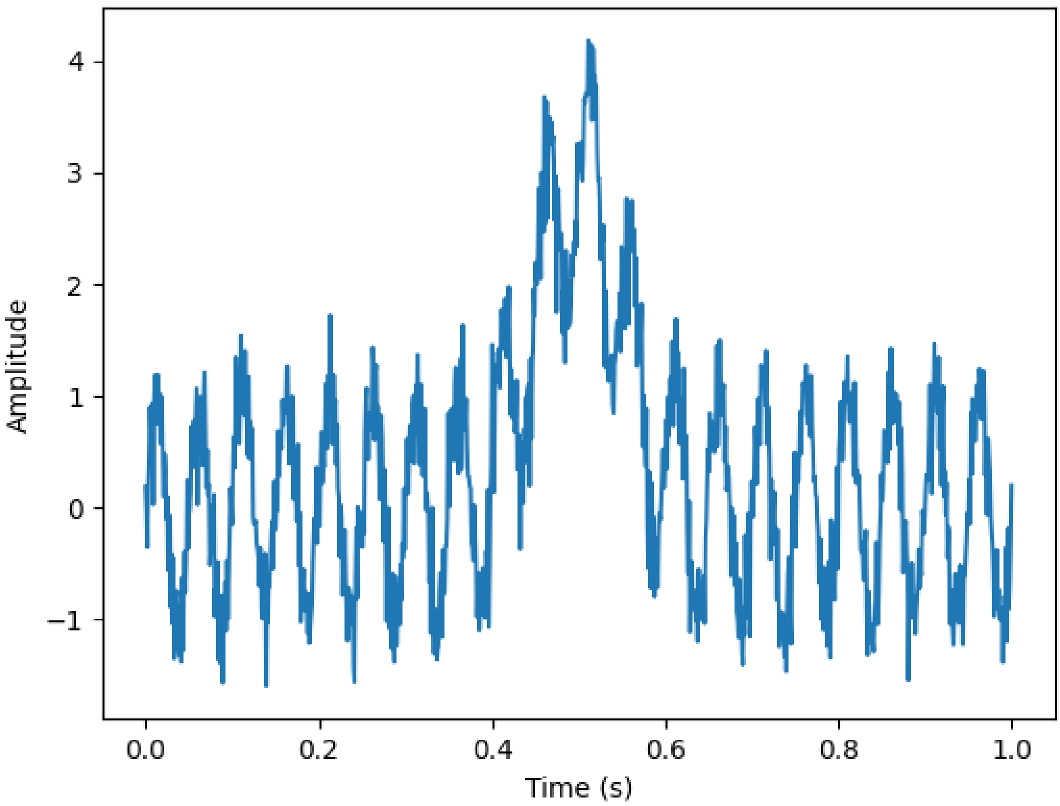

Figure 1. Impulse non-stationary signal

The signal graph clearly shows a pulse at t≈0.5t. This is a non-stationary disturbance that is difficult to detect with classical spectral methods.[7]

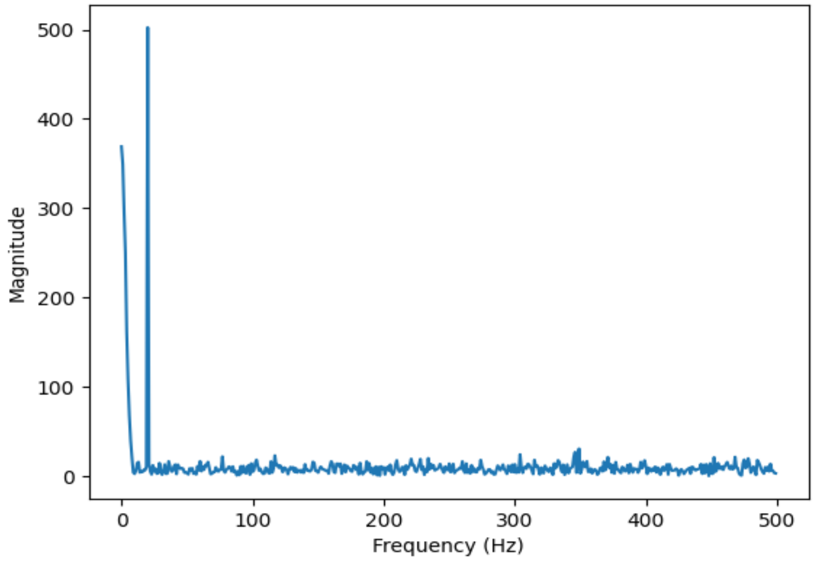

Figure 2. Fourier spectrum

The spectrum contains a fundamental frequency, but the impulse energy is spread out over the entire range. This again shows that the Fourier transform is not time-localized. [7]

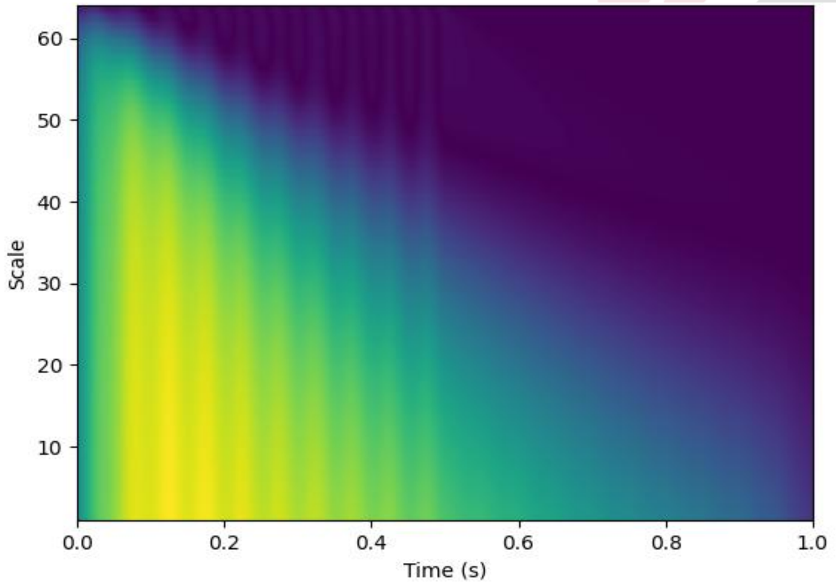

Figure 3. Wave scalogram (energy map)

The scalogram shows a multi-scale energy concentration at the pulse location. This empirically confirms the main advantage of wavelet diagnostics - time-frequency local accuracy.[8]

Conclusion

In this study, a new multiscale energy model for signal diagnostics was developed and theoretically proven based on functional analysis. It was shown that the pulse localization in wavelet space is optimal, and the approximation of the diagnostic index to the asymptotic normal distribution simplifies statistical decision-making. Numerical experiments confirmed the high sensitivity and stability of the method. This approach is expected to become an important theoretical platform for creating next-generation diagnostic algorithms for intelligent monitoring systems and industrial robots.

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