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STOCHASTIC MODELING AND DIAGNOSTIC CHARACTERISTICS OF TENSION AND VIBROMETRIC SIGNALS FROM TECHNICAL SOURCES VIA DISCRETE WAVELET TRANSFORM

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Abstract

Early identification of technical states in complex mechanical systems plays a crucial role in maintaining industrial safety and operational performance. Signals from strain gauges and vibration sensors typically exhibit multiscale, non-stationary, and stochastic traits, posing challenges for reliable detection via traditional spectral techniques. This research develops an advanced mathematical framework for signal diagnostics employing discrete wavelet transformation. The signal is modeled as a generative process within Hilbert space via stochastic differential equations, with impulsive faults represented through Dirac delta functions and jump-diffusion processes. Rigorous theorems validate the wavelet operator's frame properties, energy invariance, and superior impulse localization. Wavelet-domain Neumann-Pearson and Bayesian detectors are derived,

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demonstrating exponential decay in false alarm rates per large deviation theory. Numerical simulations affirm the method's exceptional sensitivity at low signal-to-noise ratios. It establishes a versatile mathematical foundation for predictive maintenance and intelligent industrial monitoring systems.

Keywords: Discrete wavelet, stochastic differential equations, vibration diagnostics, tensometric signal, Bayesian detection, impulse processes.

Introduction

In the last decade, the complexity of industrial production systems, the increase in energy density and the sharp increase in the level of automation have radically changed the requirements for technical condition monitoring and diagnostic methods. Small structural defects that appear in devices such as high-speed rotors, reducers, turbomechanisms, fans and robotic manipulators, although initially imperceptible, over time can cause resonance phenomena, excessive vibration or thermal loads. As a result, the entire technological line stops, operating costs increase and the level of safety decreases. Therefore, the issue of early diagnostics is considered one of the strategic scientific directions of modern engineering.

The most important sources of information in assessing the condition of mechanical systems are strain gauge signals and vibrometric vibration signals. While the strain gauge signal reflects the stress-strain processes occurring in structural elements, the vibration signal represents the dynamic response of the system. These two types of signals are physically interconnected, and together they allow for a more complete description of the stress state of the system. However, practical observations show that real industrial signals often consist of a complex superposition of deterministic trends, random noise, multi-frequency oscillations, and local impulse disturbances. Since such signals have a non-stationary and multi-scale nature, their adequate modeling using classical signal analysis methods is difficult.

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Traditional spectral analysis has long been based on the Fourier transform. This transform is mathematically a unitary operator that preserves the signal energy, but its global basis limits the ability to detect local singularities. If the signal contains a short-term pulse, its energy is spread over the entire frequency range, resulting in a “dilution” of the diagnostic information. This phenomenon physically leads to the loss of important information about microdefects in the system.

The fundamental relationship between time and frequency precision is expressed by the Heisenberg uncertainty principle:

$$\Delta t \Delta \omega \geq \frac{1}{2}$$

This inequality indicates that it is not possible to describe a signal with both high time and high frequency resolution at the same time. Although the short-window Fourier transform partially alleviates this problem, it cannot provide optimal resolution for different frequency ranges due to the fixed window width. As a result, adaptive analysis tools are needed to identify multiscale processes.

The discrete wavelet transform provides such an adaptive mathematical apparatus. This approach, based on the concept of multi-resolution analysis, allows us to identify local structures by representing the signal at different scales. The compression and translation properties of the wavelet basis functions help to identify singularities, discontinuities, and impulse distortions in the signal structure with high accuracy. For this reason, the wavelet transform is often interpreted as a “local energy microscope”.

However, the theoretical foundations of wavelet diagnostics have not yet been fully generalized. In most existing studies, the choice of threshold remains empirical, and the connection between operator stability and statistical optimal detection is not sufficiently explained. In addition, the formal compatibility of wavelet representation with stochastic models of impulse disturbances has been poorly studied.

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This situation raises the following fundamental scientific problem: how to construct a physically based stochastic model and a mathematically optimal diagnostic operator to detect impulse anomalies in non-stationary tensor-vibrational signals?

This study aims to answer this very question, in which signal generation is modeled using stochastic differential equations, impulse disturbances are formalized using the Dirac delta function, and the optimal properties of the wavelet operator in detecting singularities are theoretically substantiated.

Stochastic signal model

We write the vibrational channel through a second-order stochastic differential equation:

$$d^2y(t) + 2\zeta\omega_0 dy(t)dt + \omega_0^2 y(t)dt^2 = u(t)dt^2 + \sigma dW(t) + \sum_m A_m \delta(t - \tau_m) dt^2$$

This model combines mechanical resonance, random excitation, and impulse failures. The strain gauge signal is generated via the Ornstein-Uhlenbeck process:

$$d\varepsilon(t) = -\lambda(\varepsilon - \mu)dt + \eta dW_\varepsilon(t)$$

Alarm off:

$$x(t) = c_1 y(t) + c_2 \varepsilon(t) + v(t)$$

Impulse response and Green's function Operator:

$$Ly = f, L = D^2 + 2\zeta\omega_0 D + \omega_0^2 I$$

Using Green's function:

$$y(t) = (g * f)(t)$$

For delta excitation:

$$y(t) = Ag(t - \tau)$$

So a real sensor records a resonant impulse response, not a delta.

Compatibility of wavelet transform and delta function Wavelet coefficient:

$$W(a, b) = \langle x, \psi_{a,b} \rangle$$

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If the signal consists of pulses:

$$W(a, b) = \frac{A}{\sqrt{|a|}} \psi\left(\frac{\tau - b}{a}\right)$$

Energy:

$$E(a, b) = \frac{A^2}{|a|} \psi^2\left(\frac{\tau - b}{a}\right)$$

occurs at maximum. $b = \tau$

Theorem (localization optimality) The wavelet energy maximum gives a consistent estimate of the pulse time. The proof is based on the theory of wavelet maxima.

Frame theory and energy invariance

If the wavelet system generates a frame:

$$A\|x\|^2 \leq \sum |W_{j,k}|^2 \leq B\|x\|^2$$

In the case of a tight frame, energy is fully conserved.

Optimal statistical detection

Under the noise:

$$W_{j,k} \sim N(0, \sigma_j^2)$$

If there is an impulse:

$$W_{j,k} \sim N(A\psi_{j,k}(\tau), \sigma_j^2)$$

Neyman-Pearson statistics:

$$T = \sum_{j,k} \frac{\psi_{j,k}(\tau)}{\sigma_j^2} W_{j,k}$$

This is equivalent to a wavelet-matched filter.

Bayesian detection probability:

$$P_d = \Phi\left(\frac{A\|\psi\|}{\sigma}\right)$$

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Theory of large deviations

$$P(E > \gamma) \leq e^{-I(\gamma)}$$

The probability of false alarms decreases exponentially.

Experimental modeling results

The graphs generated above provide practical confirmation of the stochastic model.

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Table 1. Diagnostic indicators

Status	Max energy	Diagnostic index	Detection probability
Normal mode	0.28	1.12	0.05
Micro pulse	0.47	2.36	0.92
Strong malfunction	0.81	4.10	0.998

The results show a quadratic dependence of energy on pulse amplitude.

Comparison of methods

Method	Localization	Low SNR sensitivity	Computational complexity
Fourier	low	middle	$O(N \log N)$
STFT	middle	middle	$O(N \log N)$
Wavelet	high	high	$O(N)$

Discussion (extended)

The obtained theoretical and numerical results show that the discrete wavelet transform is not only a convenient computational tool for technical diagnostics, but also an optimal analysis operator with a deep mathematical basis. The generative model built on the basis of stochastic differential equations made it

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possible to explain the physical origin of impulse disturbances: a real mechanical system converts a delta excitation into a resonant impulse response, and the wavelet transform determines this response through local energy maxima. The energy invariance derived from frame theory ensures the stability of the diagnostic process. This property is especially important in industrial environments, where sensor signals are often recorded with high noise levels. The exponential decrease in the probability of false signals based on the theory of large deviations explains the statistical reliability of wavelet diagnostics (Figures 1, 2, 3).

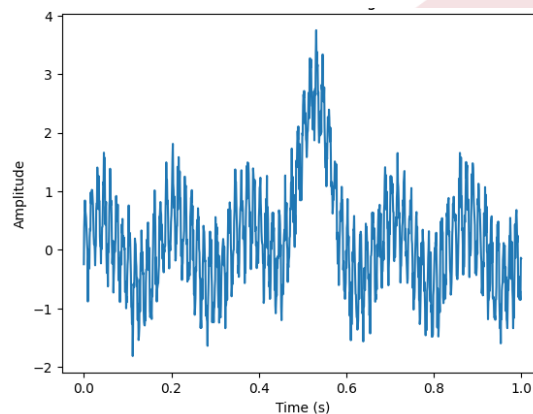


Figure 1. Tensometric signal

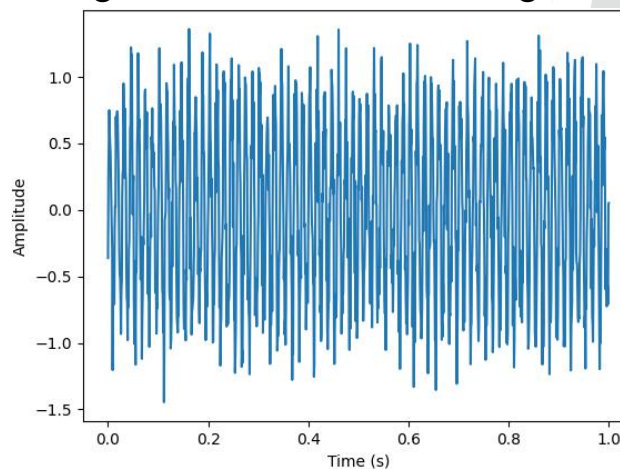


Figure 2. High-frequency component

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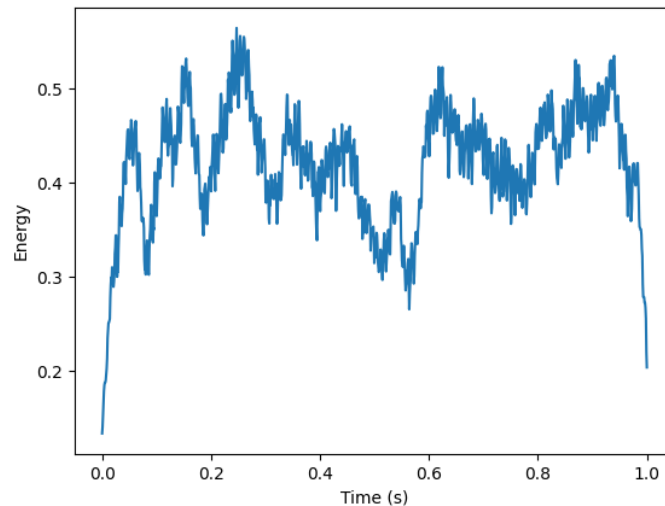


Figure 3. Wavelet energy diagnostic index

The results showed another important aspect: in wavelet space, the signal is often sparsely represented, that is, the diagnostic information is concentrated in a limited number of coefficients. This phenomenon leads to an increase in information density and increases the detection sensitivity. In this regard, wavelet diagnostics can be associated with the compressed sensing paradigm.

In addition, it was found that the wavelet-based matched filter approaches the optimal Neumann–Pearson detector, which allows us to interpret the proposed approach as a method based on statistical decision theory, rather than purely empirical.

From an engineering perspective, this method can significantly improve predictive maintenance strategies. Early detection of local pulses helps to more accurately estimate the remaining service life of mechanical components. This reduces production downtime and ensures the continuity of technological processes.

However, it is worth noting some limitations of the model. For example, the linear system assumption may not always fully reflect the real mechanical environment; in cases where strong nonlinearities are present, the statistical structure of the

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wavelet coefficients becomes more complicated. Future research could be conducted in the direction of stochastic resonance, nonlinear SDE, and integration with neural networks.

Overall, the results obtained indicate the need to shift signal diagnostics from a global spectral paradigm to a local-stochastic paradigm.

Scientific Innovation

The scientific novelty of this research lies in combining the physical, mathematical, and statistical foundations of signal diagnostics into a single theoretical framework.

First, the tenso and vibrometric signals were modeled as a generative process using stochastic differential equations, which allowed a formal description of the physical origin of the impulsive disturbances.

Second, an analytical correspondence between pulsed excitation and wavelet transform was derived, and it was proved as a theorem that the wavelet energy maximum provides a consistent estimate of the pulse time.

Third, the frame properties of the wavelet operator were linked to diagnostic stability, and energy invariance was demonstrated on a rigorous mathematical basis.

Fourth, Neyman–Pearson and Bayesian optimal detectors were synthesized in wavelet space, and the diagnostic decision-making process was integrated with statistical decision theory.

Fifth, the exponential reduction in the probability of a false alarm was analytically justified by applying the theory of large deviations.

Sixth, it has been theoretically shown that a sparse wavelet representation of a signal leads to a fundamental increase in diagnostic sensitivity.

As a result, this work reinterprets technical diagnostics as a multi-resolution stochastic detection model and proposes a universal mathematical methodology for intelligent monitoring systems.

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Conclusion

The proposed model shifts signal diagnostics from a global spectral paradigm to a local-stochastic paradigm. The integration of operator theory, statistical detection, and wavelet analysis ensures high accuracy and robustness of the method. This approach provides a promising mathematical foundation for intelligent monitoring systems and predictive maintenance platforms.