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INTERLAYER SHIFTS OF TWO-LAYER COMPOSITE REINFORCED PLATES AND SHELLS TAKING INTO ACCOUNT TEMPERATURE LOADS

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Abstract

Combined plates and shells consisting of two layers interconnected by a pliable thin adhesive seam under the influence of external temperature loads are considered. It is found that the shear modulus and the thickness of the seam have a great influence on the strength and deformability of combined double-layer shells, especially during thermal heating.

Keywords: Combined two-layer plate and shell, interlayer shifts, temperature load, thickness of the bearing and reinforcing layer, shear stresses, shear modulus, shear function.

The combined plates and shells consisting of two layers interconnected by a pliable thin adhesive seam under the influence of external temperature loads are considered. The coordinate system is assumed according to Fig.1. The stress-strain state of the combined shells will be determined under the following assumptions:

- 1) The thicknesses of the orthotropic layers are constant, and the shell and plate work only in the elastic stage.;
- 2) the thickness of the bearing layer is significantly greater than the reinforcing layer ($h > \delta$);

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- 3) tangential stresses $\tau_{\alpha\gamma}$, $\tau_{\beta\gamma}$ or the corresponding deformations $e_{\alpha\gamma}$, $e_{\beta\gamma}$ the thickness of the shell varies according to a given law
- 4) The mixing normal to the median surface of the shell is independent of the coordinate γ [1];
- 5) there is no pressure between the layers ($\sigma_\gamma = 0$).

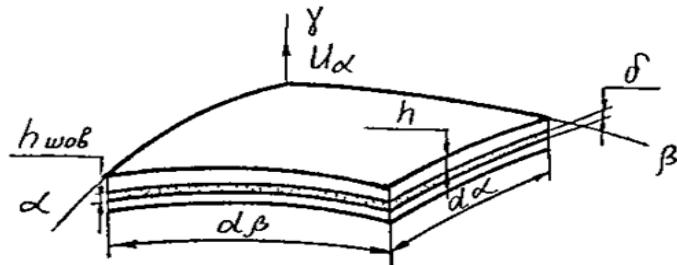


Fig.1 Combined two-layer shell

Let's also assume that there is a thin adhesive layer between the two load-bearing and reinforcing layers, which works only for vertical shear. The adhesive layer does not accept either tensile or bending stresses. The tangential stresses acting in this layer are transmitted to the bearing and reinforcing layers. The law of the distribution of these stresses in the layers can be assumed to be linear so that the boundary conditions for tangential stresses on the upper and lower surfaces are satisfied.

In this regard, tangential stresses have the following analytical expressions:

a) in the carrier (first) layer

$$\tau_{\alpha\gamma,(\beta\gamma)} = \tau_{1,(2)}(\alpha, \beta) \left(\frac{1}{2} + \frac{\gamma}{h} \right); \quad (1)$$

b) in the reinforcing layer

$$\tau_{\alpha\gamma,(\beta\gamma)} = \tau_{1,(2)}(\alpha, \beta) \left(\frac{1}{2} + \frac{\gamma_1}{\delta_n} \right); \quad (2)$$

Taking into account the accepted hypotheses, we have:

$$e_\gamma = 0; U_\gamma = \omega(\alpha, \beta); \quad (3)$$

Where h, δ – thickness of the bearing and reinforcing layers;

$\Phi_i = \Phi_i(\alpha, \beta)$ - arbitrary desired shift functions;

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$\tau_i = \tau_i(\alpha, \beta)$ - desired tangential stresses;

$G_{ik}^{(1)}, G_{ik}^{(2)}$ - the shear modules of the first and second layers ($i=1,2; K=3$).

The coordinates of γ have the following change boundaries: for the first layer

$$-\frac{h}{2} \leq \gamma \leq +\frac{h}{2}; \text{ for the second one } -\frac{\delta}{2} \leq \gamma_1 \leq +\frac{\delta}{2}.$$

When solving this problem, the hypotheses mentioned in [1,2] are valid... The construction of a refined theory in the work is based on energy considerations[1,6]

It is believed that the heat flow acts in the transverse direction. From the solution of the thermal conductivity problem, the following temperature distribution in the layers is obtained: in the first layer $T_1 = T_1^0 + \theta_1 \gamma$ in the second layer $T_2 = T_2^0 + \theta_z \gamma_1$

At the same time $-\frac{h}{2} \leq \gamma \leq +\frac{h}{2}$;

θ_1, θ_z - Temperature gradients in the layers;

T_1^0, T_2^0 - Temperatures of the median planes of the layers.

Taking as usual for full deformations:

$$\varepsilon_{noi} = \varepsilon^y + \varepsilon_T, \quad (4)$$

Where: ε^y - elastic deformations of the system,

ε^T - deformation of layers due to temperature loads,

To obtain the basic equilibrium equations of combined two-layer elastic shells with malleable seams, we use the Lagrange variational principle, which opens a natural way to reduce the three-dimensional problem of continuum mechanics to one-dimensional and two-dimensional problems. [4,5,7]

It should be noted that the principle under consideration simultaneously allows us to obtain appropriate natural boundary conditions for each selected unknown, and also serves as the basis for various approximate methods, including for solving combined orthotropic plates and shells with interlayer shifts.

An expression full of energy

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$$U = \frac{1}{2} \iiint (\sigma_{\alpha}^{(i)} \varepsilon_{\alpha}^{(i)} + \sigma_{\beta}^{(i)} \varepsilon_{\beta}^{(i)} + \tau_{\alpha\beta}^{(i)} \varepsilon_{\alpha\beta}^{(i)} + \tau_{\alpha\gamma}^{(i)} \varepsilon_{\alpha\gamma}^{(i)} + \tau_{\beta\gamma}^{(i)} \varepsilon_{\beta\gamma}^{(i)}) d\nu + \\ + \frac{1}{2} \iint (\tau_1^{III} \varepsilon_{w13} + \tau_2^{III} \varepsilon_{w23} - 2qW) ds \quad (5)$$

Integrating (5) over the thickness (from $-h/2$ to $+h/2$ and for the second layer $-\delta n/2$ to $+\delta n/2$), we obtain an expression containing unknown functions and their derivatives

$$U = 1/2 \iint \left\{ U_F \left((d^2 W)/(d\alpha^2), (d^2 W)/(d\beta^2), (d^2 W)/d\alpha d\beta, (du_o)/d\alpha, (du_o)/d\beta, (d\phi_1)/d\alpha, (d\phi_1)/d\beta, (d\phi_2)/d\alpha, (d\phi_2)/d\beta, (d\tau_1)/d\alpha, (d\tau_1)/d\beta, (d\tau_2)/d\alpha, (d\tau_2)/d\beta, dW/d\alpha, dW/d\beta, u_o, V_o, \phi_1, \phi_2, \tau_1, \tau_2, W \right) \right\} d\alpha, d\beta \quad (6)$$

According to the Lagrange variational principle, the potential energy of an elastic system in the equilibrium position assumes a stationary value. It consists of the potential energy of elastic deformation of the layers, the adhesive seam and the work of the external load (5). The integrals of the first sum on the right side of the formula (5), after thickness integration, are taken over the area of the median surface Ω , the remaining integrals along the contour S . According to this principle, in the equilibrium position, the first variation from the total potential energy of the system must be equal to zero: $\delta E=0$.

As an example, to analyze the effect of the ductility of the adhesive layer on the VAT of a two-layer shell, taking into account the transverse shifts, we take the following measurement. The calculation is performed with the following parameters.

$$E_I^{(1)} = E_2^{(1)} = 2.02 \cdot 10^5; \mu_{12}^{(1)} = \mu_{21}^{(1)} = 0.285; E_I^{(2)} = 0.471 \cdot 10^5 \text{ Mpa} \\ E_I^{(2)} = 0.49 \cdot 10^5 \text{ Mpa}, G_{12}^1 = G_{13}^1 = G_{23}^1 = 7.87 \cdot 10^4 \text{ Mpa}; G_{12}^2 = 5.5 \cdot 10^3 \text{ Mpa}, G_{13}^2 = 4.2 \cdot 10^3 \text{ Mpa}, G_{23}^2 = 0.35 \cdot 10^3 \text{ Mpa}, \mu_{12}^{(2)} = \mu_{21}^{(2)} + 0.385, R_M = 10.40 \text{ sm}, R_n = 10.30 \text{ sm};$$

Where τ_1 – tangential stress of the first bearing layer,

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W - deflection,

If uneven heating is taken into account at the same time ($T_{\text{нап}} = 20^{\circ}\text{C}$, $T_{\text{BH}} = 200^{\circ}\text{C}$, $T_{\text{шов}} = 68^{\circ}\text{C}$, $q = 0$), that increase $G_{\text{шк}}$ 100 times, from $G_{\text{шк}} = 5 \cdot 10^2 \text{ Mpa}$ up to $5 \cdot 10^4 \text{ Mpa}$ leads to

- a) to reduce deflection by 6.4%;
- б) reducing U_0 to 2,7%;
- в) reduction τ_1 to 24%;
- г) reduction ϕ_1 to 45%;

The examples shown above have shown that the shear modulus and the thickness of the seam have a great effect on the strength and deformability of combined two-layer shells if the shear modulus of the bonding layer is significantly less than the shear modulus of the layers.

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