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# DEFORMATION AND BOUNDARY CONDITIONS OF COMPOSITE REINFORCED CONCRETE SLABS.

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### Abstract

The paper is devoted to the study of the stress-strain state (SSS) of reinforced concrete composite-reinforced double-layer combined slabs. The derivation of equations and formulation of boundary conditions are given, taking into account the interlayer shear and the compliance of the adhesive joint. The analysis of the influence of the geometric and physical parameters of double-layer combined slabs on the SSS is presented.

### Introduction

We assume that the two-layer combined composite-reinforced reinforced concrete slab under consideration consists of a bearing layer and a reinforcing layer. We assume that:

1. The thicknesses of the first load-bearing (reinforced concrete), second (composite) reinforcing and bonding (connecting) layers are constant.
2. The thickness of the load-bearing (thick) layer is significantly greater than that of the second reinforcing layer. ( $h > \delta_n$ ).
3. In the case of slabs, the accepted hypotheses based on the refined theory of S.A. Ambartsumian are valid. [1, 2]:

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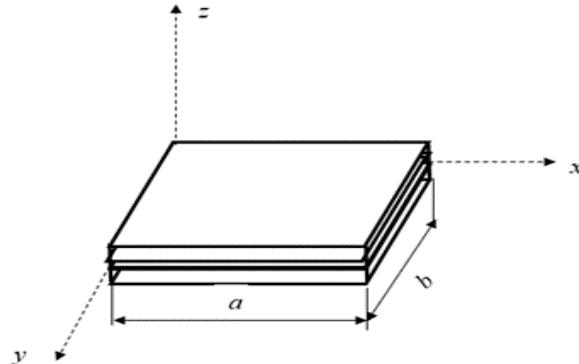
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Fig. 1. Composite-reinforced double-layer combined reinforced concrete slab.

Assuming the hypotheses of S.A. Ambartsumian [1, 2] approximately, we consider that the relative elongation of the deformation in the Z direction is equal to zero.

Shear deformations of the first layer.

$$\begin{aligned}\ell_{xz}^{(1)} &= 0,5 \left( \frac{h^2}{4} - \gamma^2 \right) \Phi_1 + \left( 0,5 - \frac{\gamma}{h} \right) \frac{\tau_1}{G_{13}^{(1)}} \\ \ell_{yz}^{(1)} &= 0,5 \left( \frac{h^2}{4} - \gamma^2 \right) \Phi_2 + \left( 0,5 - \frac{\gamma}{h} \right) \frac{\tau_2}{G_{23}^{(1)}}\end{aligned}\quad (1)$$

Similarly, the shear deformations of the second composite reinforcement layer vary with the thickness of the slab according to the following specified law

$$\begin{aligned}\ell_{xz}^{(2)} &= \left( 0,5 + \frac{\gamma_1}{\delta_n} \right) \frac{\tau_1}{G_{13}^{(2)}} \\ \ell_{yz}^{(2)} &= \left( 0,5 + \frac{\gamma_1}{\delta_n} \right) \frac{\tau_2}{G_{23}^{(2)}}\end{aligned}\quad (2)$$

$\Phi_i = \Phi_i(x, y)$  - arbitrary desired shift functions;

$\tau_i = \tau_i(x, y)$  - desired tangential stresses;

$G_{i,K}^{(1)}, G_{i,K}^{(2)}$  - the shear modules of the first and second layers ( $i=1, 2; K=3$ ).

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To obtain the basic deformation equations for two-layer composite reinforced concrete slabs, we use the Lagrangian variational principle, taking into account the shear rigidity.

We will determine the potential energy of an elastic reinforced concrete slab using the components of the stress and strain tensors and present it in the following form:

$$\begin{aligned}
 U(z) = & \frac{1}{2} \int_s \iint_{-\frac{h}{2}}^{\frac{h}{2}} \left( \sigma_x^{(1)} \varepsilon_x^{(1)} + \sigma_y^{(1)} \varepsilon_y^{(1)} + \tau_{xy}^{(1)} \varepsilon_{xy}^{(1)} + \tau_{xz}^{(1)} \times \gamma_{xz}^{(1)} + \tau_{yz} \times \gamma_{yz} \right) \\
 & \times d_x d_y d_y \\
 & + \int_s \iint_{-\frac{\delta_n}{2} - \frac{h_2}{2}}^{\frac{\delta_n}{2} + \frac{h_2}{2}} \left( \sigma_x^{(2)} \varepsilon_x^{(2)} + \sigma_y^{(2)} \varepsilon_y^{(2)} + \tau_{xy}^{(2)} \varepsilon_{xy}^{(2)} + \tau_{xz}^{(2)} \times \gamma_{xz}^{(2)} + \tau_{yz}^{(2)} \times \right. \\
 & \left. \gamma_{yz}^{(2)} \right) d_x d_y d_y + \frac{1}{2} \iint (\tau_1^w \varepsilon_{w13} + \tau_2^w \varepsilon_{w23} + \tau_3^2 \varepsilon_{w13} + \tau_4^2 \varepsilon_{w23} 2qw) ds. \\
 (3)
 \end{aligned}$$

Integrating by thickness (for the first layer –from  $-\frac{h}{2}$  before  $+\frac{h}{2}$ , for the second one, from  $-\frac{\delta_n}{2}$  до  $\frac{\delta_n}{2}$ ) we will obtain functional expressions in the form of a double integral

$$\begin{aligned}
 U = & \frac{1}{2} \iint U_F \left( \frac{\partial u_0}{\partial x}, \frac{\partial u_0}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial \Phi_1}{\partial x}, \frac{\partial \Phi_1}{\partial y}, \frac{\partial \Phi_2}{\partial x}, \frac{\partial \Phi_2}{\partial y}, \frac{\partial \tau_1}{\partial x}, \frac{\partial \tau_1}{\partial y}, \frac{\partial \tau_2}{\partial x}, \right. \\
 & \left. \frac{\partial \tau_2}{\partial y}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial x \partial y}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, U_0, V_0, \Phi_1, \Phi_2, \tau_1, \tau_2, w \right) ds \\
 (4)
 \end{aligned}$$

For this functional, the well-known Euler variational equation is as follows:

$$F_z - \frac{\partial}{\partial x} \left\{ F_{\frac{\partial z}{\partial x}} \right\} - \frac{\partial}{\partial y} \left\{ F_{\frac{\partial z}{\partial y}} \right\} + \frac{\partial}{\partial x^2} \left\{ F_{\frac{\partial^2 z}{\partial x^2}} \right\} + \frac{\partial^2}{\partial x \partial y} \left\{ F_{\frac{\partial^2 z}{\partial x \partial y}} \right\} + \frac{\partial^2}{\partial y^2} \left\{ F_{\frac{\partial^2 z}{\partial y^2}} \right\} = 0 \quad (5)$$

After some transformations, boundary conditions must be added to this equation, which are obtained using the Green's formula.

For W, they can be obtained based on the following expressions

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$$\begin{aligned}
 & \oint_c \left[ F_{\frac{\partial^2 w}{\partial x^2}} \delta \left( \frac{\partial w}{\partial x} \right) dy - F_{\frac{\partial^2 w}{\partial y^2}} \delta \left( \frac{\partial w}{\partial y} \right) dx \right] - \oint_c \left[ \frac{\partial}{\partial x} \left\{ F_{\frac{\partial^2 w}{\partial x^2}} \right\} + \right. \\
 & \left. + \frac{\partial}{\partial y} \left\{ F_{\frac{\partial^2 w}{\partial x \partial y}} \right\} \right] \delta w dy + \oint_c \left[ \frac{\partial}{\partial x} \left\{ F_{\frac{\partial^2 w}{\partial x \partial y}} \right\} + \frac{\partial}{\partial x} \left\{ F_{\frac{\partial^2 w}{\partial y^2}} \right\} \right] \times \\
 & \times \delta w dx + \oint_c \left( F_{\frac{\partial w}{\partial x}} dy - F_{\frac{\partial w}{\partial y}} dx \right) \delta w = 0 \quad (6)
 \end{aligned}$$

The remaining boundary conditions for  $U_0, V_0, \Phi_1, \Phi_2, \tau_1$  и  $\tau_2$  they have the following form:

$$\begin{aligned}
 & \oint_c \left( F_{\frac{\partial u_0}{\partial x}} \cos(\hat{y}s) - F_{\frac{\partial u_0}{\partial y}} \cos(\hat{x}s) \right) \delta U_0 = 0; \\
 & \oint_c \left( F_{\frac{\partial v_0}{\partial x}} \cos(\hat{y}s) - F_{\frac{\partial v_0}{\partial y}} \cos(\hat{x}s) \right) \delta V_0 = 0; \\
 & \oint_c \left( F_{\frac{\partial \Phi_{1,(2)}}{\partial x}} \cos(\hat{y}s) - F_{\frac{\partial \Phi_{1,(2)}}{\partial y}} \cos(\hat{x}s) \right) \delta \Phi_{1,(2)} = 0; \quad (7) \\
 & \oint_c \left( F_{\frac{\partial \Phi_{1,(2)}}{\partial x}} \cos(\hat{y}s) - F_{\frac{\partial \Phi_{1,(2)}}{\partial y}} \cos(\hat{x}s) \right) \delta \tau_{1,(2)} = 0.
 \end{aligned}$$

For  $U_0, V_0, \Phi_1, \Phi_2, \tau_1$  и  $\tau_2$  Natural boundary conditions can be obtained in a similar way. The integrand functions in formulas (7) take the following expressions

$$\begin{aligned}
 F_{\frac{\partial u_0}{\partial x}} = & 2K_{1u_0} \frac{\partial u_0}{\partial x} + K_{3u_0} \frac{\partial v_0}{\partial y} + K_{4\tau_1} \frac{\partial \tau_1}{\partial x} + K_{4\tau_2} \frac{\partial \tau_2}{\partial y} + \\
 & + K_{9u_0} \frac{\partial^2 w}{\partial x^2} + K_{10u_0} \frac{\partial^2 w}{\partial y^2} + K_{14\Phi} \frac{\partial \Phi_1}{\partial x} + K_{15\Phi_1} \frac{\partial \Phi_2}{\partial y};
 \end{aligned}$$



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$$\begin{aligned} F_{\frac{\partial u_0}{\partial y}} &= 2K_{2u_0} \frac{\partial u_0}{\partial y} + K_{4u_0} \frac{\partial V_0}{\partial x} + K_{5\tau_2} \frac{\partial \tau_2}{\partial x} + K_{5\tau_1} \frac{\partial \tau_1}{\partial y} + \\ &\quad + K_{12\Phi_1} \frac{\partial \Phi_2}{\partial x} + K_{12\Phi_1} \frac{\partial \Phi_1}{\partial y} + K_{15u_0} \frac{\partial^2 w}{\partial x \partial y}; \\ F_{\frac{\partial v_0}{\partial x}} &= 2K_{2u_0} \frac{\partial v_0}{\partial x} + K_{4u_0} \frac{\partial U_0}{\partial y} + K_{5\tau_2} \frac{\partial \tau_2}{\partial x} + K_{5\tau_1} \frac{\partial \tau_1}{\partial y} + \\ &\quad + K_{12\Phi_1} \frac{\partial \Phi_2}{\partial x} + K_{12\Phi_1} \frac{\partial \Phi_1}{\partial y} + K_{15u_0} \frac{\partial^2 w}{\partial x \partial y}; \\ F_{\frac{\partial v_0}{\partial y}} &= 2K_{2v_0} \frac{\partial v_0}{\partial y} + K_{3u_0} \frac{\partial u_0}{\partial x} + K_{3\tau_2} \frac{\partial \tau_2}{\partial y} + K_{3\Phi_2} \frac{\partial \Phi_2}{\partial x} + \\ &\quad + K_{10v_0} \frac{\partial^2 w}{\partial x^2} + K_{10v_0} \frac{\partial^2 w}{\partial y^2} + K_{15\Phi_1} \frac{\partial \Phi_1}{\partial x} + K_{15\Phi_2} \frac{\partial \Phi_2}{\partial y}; \\ F_{\frac{\partial \Phi_1}{\partial x}} &= 2K_{1\Phi_1} \frac{\partial \Phi_1}{\partial x} + K_{3\Phi_1} \frac{\partial^2 w}{\partial x^2} + K_{4\Phi_1} \frac{\partial^2 w}{\partial y^2} + K_{9\tau_1} \frac{\partial \tau_1}{\partial x} + \\ &\quad + K_{10\tau_1} \frac{\partial \tau_2}{\partial y} + K_{8\Phi_1} \frac{\partial \Phi_2}{\partial y} + K_{14\Phi_1} \frac{\partial U_0}{\partial x} + K_{15\Phi_1} \frac{\partial V_0}{\partial y}; \\ F_{\frac{\partial \Phi_1}{\partial y}} &= 2K_{2\Phi_1} \frac{\partial \Phi_1}{\partial y} + K_{5\Phi_1} \frac{\partial^2 w}{\partial x \partial y} + 2K_{2\Phi_1} \frac{\partial \Phi_2}{\partial x} + K_{11\tau_2} \frac{\partial \tau_2}{\partial x} + \\ &\quad + K_{11\tau_1} \frac{\partial \tau_1}{\partial y} + K_{12\Phi_1} \frac{\partial v_0}{\partial x} + K_{12\Phi_1} \frac{\partial U_0}{\partial y}; \\ F_{\frac{\partial \Phi_2}{\partial x}} &= 2K_{2\Phi_1} \frac{\partial \Phi_2}{\partial x} + K_{5\Phi_1} \frac{\partial^2 w}{\partial x \partial y} + 2K_{2\Phi_1} \frac{\partial \Phi_2}{\partial y} + K_{11\tau_2} \frac{\partial \tau_2}{\partial x} + \\ &\quad + K_{12\tau_1} \frac{\partial \tau_1}{\partial y} + K_{12\Phi_1} \frac{\partial V_0}{\partial x} + K_{12\Phi_1} \frac{\partial U_0}{\partial y}; \\ F_{\frac{\partial \Phi_2}{\partial y}} &= 2K_{2\Phi_2} \frac{\partial \Phi_2}{\partial y} + K_{4\Phi_1} \frac{\partial^2 w}{\partial x^2} + K_{4\Phi_2} \frac{\partial^2 w}{\partial y^2} + K_{10\tau_1} \frac{\partial \tau_1}{\partial x} + \\ &\quad + K_{10\tau_2} \frac{\partial \tau_2}{\partial y} + K_{8\Phi_1} \frac{\partial \Phi_1}{\partial x} + K_{15\Phi_1} \frac{\partial u_0}{\partial x} + K_{15\Phi_2} \frac{\partial v_0}{\partial y}; \end{aligned}$$

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$$\begin{aligned}
 F_{\frac{\partial \tau_1}{\partial x}} &= 2K_{1\tau_1} \frac{\partial \tau_1}{\partial x} + K_{3\tau_1} \frac{\partial v_0}{\partial y} + K_{4\tau_1} \frac{\partial u_0}{\partial x} + K_{7\tau_1} \frac{\partial^2 w}{\partial x^2} + \\
 &\quad + K_{8\tau_1} \frac{\partial^2 w}{\partial y^2} + K_{9\tau_1} \frac{\partial \Phi_1}{\partial x} + K_{10\tau_1} \frac{\partial \Phi_2}{\partial y} + K_{13\tau_1} \frac{\partial \tau_2}{\partial y}; \\
 F_{\frac{\partial \tau_1}{\partial y}} &= 2K_{2\tau_1} \frac{\partial \tau_1}{\partial y} + K_{5\tau_1} \frac{\partial v_0}{\partial x} + K_{6\tau_1} \frac{\partial u_0}{\partial y} + K_{11\tau_1} \frac{\partial \Phi_1}{\partial y} + \\
 &\quad + K_{11\tau_1} \frac{\partial \Phi_2}{\partial x} + K_{14\tau_1} \frac{\partial \tau_2}{\partial x} + K_{15\tau_1} \frac{\partial^2 w}{\partial x \partial y}; \\
 F_{\frac{\partial \tau_2}{\partial x}} &= 2K_{1\tau_2} \frac{\partial \tau_2}{\partial x} + K_{5\tau_1} \frac{\partial v_0}{\partial x} + K_{5\tau_1} \frac{\partial u_0}{\partial y} + K_{11\tau_2} \frac{\partial \Phi_1}{\partial y} + \\
 &\quad + K_{12\tau_2} \frac{\partial \Phi_2}{\partial x} + K_{14\tau_2} \frac{\partial \tau_1}{\partial y} + K_{15\tau_2} \frac{\partial^2 w}{\partial x \partial y}; \\
 F_{\frac{\partial \tau_2}{\partial y}} &= 2K_{2\tau_2} \frac{\partial \tau_2}{\partial y} + K_{3\tau_2} \frac{\partial v_0}{\partial y} + K_{3\tau_1} \frac{\partial u_0}{\partial x} + K_{7\tau_2} \frac{\partial^2 w}{\partial y^2} + \\
 &\quad + K_{8\tau_1} \frac{\partial^2 w}{\partial x^2} + K_{9\tau_2} \frac{\partial \Phi_1}{\partial x} + K_{10\tau_2} \frac{\partial \Phi_2}{\partial y} + K_{13\tau_2} \frac{\partial \tau_1}{\partial x}; \tag{8}
 \end{aligned}$$

Let's consider the resulting system of differential equations for a combined composite reinforced concrete slab:

$$\begin{aligned}
 &K_{9u_0} \frac{\partial^3 w}{\partial x^3} + 2U_4 \frac{\partial^3 w}{\partial x \partial y} + 2K_{1u_0} \frac{\partial^2 u_0}{\partial x^2} + 2K_{2u_0} \frac{\partial^2 U_0}{\partial y^2} + \\
 &+ K_{4\tau_1} \frac{\partial^2 \tau_1}{\partial x^2} + K_{5\tau_1} \frac{\partial^2 \tau_1}{\partial y^2} + T_{23} \frac{\partial^2 \tau_2}{\partial x \partial y} + U_3 \frac{\partial^2 v_0}{\partial x \partial y} + F_5 \times \\
 &\times \frac{\partial^2 \Phi_2}{\partial x \partial y} + K_{14\Phi_1} \frac{\partial^2 \Phi_1}{\partial x^2} + K_{12\Phi_1} \frac{\partial^2 \Phi_1}{\partial y^2} = 0 \\
 &K_{10v_0} \frac{\partial^3 w}{y^3} + 2U_4 \frac{\partial^3 w}{\partial x^2 \partial y} + 2K_{1v_0} \frac{\partial^2 V_0}{\partial x^2} + 2K_{2v_0} \frac{\partial^2 V_0}{\partial y^2} + \\
 &+ K_{5\tau_1} \frac{\partial^2 \tau_2}{\partial x^2} + K_{3\tau_2} \frac{\partial^2 \tau_2}{\partial y^2} + T_3 \frac{\partial^2 \tau_1}{\partial x \partial y} + U_3 \frac{\partial^2 U_0}{\partial x \partial y} +
 \end{aligned}$$



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$$\begin{aligned} & +F_5 \frac{\partial^2 \Phi_1}{\partial x \partial y} + K_{12\Phi_1} \frac{\partial^2 \Phi_2}{\partial x^2} + K_{15\Phi_2} \frac{\partial^2 \Phi_2}{\partial y^2} = 0; \\ & K_{3\Phi_1} \frac{\partial^3 w}{\partial x^3} + 2F_3 \frac{\partial^3 w}{\partial x \partial y^2} + 2K_{1\Phi_1} \frac{\partial^2 \Phi_1}{\partial x^2} + 2K_{2\Phi_1} \frac{\partial^2 \Phi_1}{\partial y^2} + K_{9\tau_1} \times \\ & \times \frac{\partial^2 \tau_1}{\partial x^2} + K_{11\tau_1} \frac{\partial^2 \tau_1}{\partial y^2} + T_{25} \frac{\partial^2 \tau_2}{\partial x \partial y} + F_5 \frac{\partial^2 V_0}{\partial x \partial y} + F_4 \times \\ & \times \frac{\partial^2 \Phi_2}{\partial x \partial y} + K_{14\Phi_1} \frac{\partial^2 U_0}{\partial x^2} + K_{12\Phi_1} \frac{\partial^2 u_0}{\partial y^2} - 2K_{16\Phi_1} \Phi_1 - \frac{h^3}{12} \tau_1 = 0; \\ & K_{4\Phi_2} \frac{\partial^3 w}{\partial y^3} + 2F_3 \frac{\partial^3 w}{\partial x^2 \partial y} + 2K_{1\Phi_2} \frac{\partial^2 \Phi_2}{\partial x^2} + 2K_{2\Phi_2} \frac{\partial^2 \Phi_2}{\partial y^2} + \\ & + K_{11\tau_2} \frac{\partial^2 \tau_2}{\partial x^2} + K_{10\tau_2} \frac{\partial^2 \tau_2}{\partial y^2} + T_5 \frac{\partial^2 \tau_1}{\partial x \partial y} + T_5 \frac{\partial^2 U_0}{\partial x \partial y} + \\ & + F_4 \frac{\partial^2 \Phi_1}{\partial x \partial y} + K_{12\Phi_1} \frac{\partial^2 V_0}{\partial x^2} + K_{15\Phi_2} \frac{\partial^2 V_0}{\partial y^2} - 2K_{16\Phi_2} - \frac{h^3}{12} \tau_2 = 0; \\ & K_{7\tau_1} \frac{\partial^3 w}{\partial x^3} + 2T_4 \frac{\partial^3 w}{\partial x \partial y^2} + 2K_{1\tau_1} \frac{\partial^2 \tau_1}{\partial x^2} + 2K_{2\tau_1} \frac{\partial^2 \tau_1}{\partial y^2} + K_{9\tau_1} \frac{\partial^2 \Phi_1}{\partial x^2} + \\ & + K_{11} \frac{\partial^2 \Phi_1}{\partial y^2} + T_6 \frac{\partial^2 \tau_2}{\partial x \partial y} + T_3 \frac{\partial^2 V_0}{\partial x \partial y} + T_5 \frac{\partial^2 \Phi_2}{\partial x \partial y} + K_{4\tau_1} \frac{x^2 U_0}{\partial x^2} + \\ & + K_{5\tau_1} \frac{\partial^2 U_0}{\partial y^2} - 2K_{16\tau_1} \tau_1 - \frac{h^3}{12} \Phi_1 = 0; \\ & K_{7\tau_2} \frac{\partial^3 w}{\partial y^3} + 2T_{24} \frac{\partial^3 w}{\partial x^2 \partial y} + 2K_{1\tau_2} \frac{\partial^2 \tau_2}{\partial x^2} + 2K_{2\tau_2} \frac{\partial^2 \tau_2}{\partial y^2} + K_{12\tau_2} \frac{\partial^2 \Phi_2}{\partial x^2} + \\ & + K_{10\tau_2} \frac{\partial^2 \Phi_2}{\partial y^2} + T_{26} \frac{\partial \tau_1}{\partial x \partial y} + T_{23} \frac{\partial^2 V_0}{\partial x \partial y} + T_{25} \frac{\partial^2 \Phi_1}{\partial x \partial y} + \\ & + K_{5\tau_2} \frac{\partial^2 U_0}{\partial x^2} - K_{3\tau_2} \frac{\partial_2 V_0}{\partial y^2} - 2K_{16\tau_2} \tau_2 - \frac{h^3}{12} \Phi_2 = 0; \\ & K_{1w} \frac{\partial^4 w}{\partial x^4} + K_{3w} \frac{\partial^4 w}{\partial y^4} + K_{w_3} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{1}{2} K_{7\tau_1} \frac{\partial^3 \tau_1}{\partial x^3} + \end{aligned}$$

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$$\begin{aligned}
 & + \frac{1}{2} K_{7\tau_2} \frac{\partial^3 \tau_2}{\partial y^3} + T_4 \frac{\partial^3 \tau_1}{\partial x \partial y^2} + T_{24} \frac{\partial^3 \tau_2}{\partial x^2 \partial y} + \frac{1}{2} K_{3\Phi_1} \frac{\partial^3 \Phi_1}{\partial y^3} + \\
 & + \frac{1}{2} K_{4\Phi_2} \frac{\partial^3 \Phi_2}{\partial y^3} + F_3 \frac{\partial^3 \Phi_1}{\partial x \partial y^2} + F_3 \frac{\partial^3 \Phi_2}{\partial x^2 \partial y} + \frac{1}{2} K_{9u_0} \frac{\partial^3 U_0}{\partial x^3} + \\
 & + \frac{1}{2} K_{10v_0} \frac{\partial^3 V_0}{\partial y^3} + U_4 \frac{\partial^3 U_0}{\partial x \partial y^2} + U_4 \frac{\partial^3 V_0}{\partial y \partial x^2} - q = 0; \tag{9}
 \end{aligned}$$

The coefficients of the system of differential equations (9) are given in the appendix of [3.4].

The resulting equations are assumed to be  $\delta_n = 0$ ,  $\tau_1 = \tau_2 = 0$  they are transformed into the well-known equations for a single-layer plate obtained by S.A. Abartsumyan[2]. In this case, the shear function adopted in the present work is related to the shear function in [2] by the following equation:

$$\Phi_1^{(i)} = \Phi_1^{(i)}(x, y) G_{13}^{(i)}, \widetilde{\Phi_2^{(i)}} = \Phi_2^{(i)}(x, y) G_{23}^{(i)}.$$

It is obvious that the Sophie-Germain equation for an isotropic single-layer slab is a special case of these equations.

The given system of differential equations with boundary conditions (8,9) allows to estimate the deformability of a combined composite reinforced concrete slab with sufficient accuracy. It also makes it possible to analyze the influence of the interlayer shear of the adhesive joint and other mechanical characteristics of combined reinforced concrete slabs on the stress-strain state.

The obtained differential equations (9) for two-layer plates are a generalization of the equations obtained by S.A. Pelekh. Obviously, if we do not take into account the tangential stresses in the bonding layer, equation (9) will turn into the well-known system of differential equations obtained by S.A. Ambartsumian.[1-9].

To analyze the influence of the mentioned factors, hinged-supported slabs are considered under uniformly distributed and linearly distributed static loads.

As examples, combined slabs made on the basis of concrete and fiberglass are selected.



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As an example, the calculation of a two-layer composite reinforced concrete slab loaded in the middle by a concentrated load is performed, the left support is fixed, and the right support is hinged along the contour.

$$E_1^E = 1,08 \cdot 10^5 \text{ kg/sm}^2, E_2^E = 0,81 \cdot 10^5 \text{ kg/sm}^2, E_1^{\Pi} = 3,05 \cdot 10^5 \text{ kg/sm}^2, \\ E_2^{\Pi} = 1,88 \cdot 10^5 \frac{\text{kg}}{\text{sm}^2}$$

-Poisson coefficients of concrete and fiberglass  $\mu_{12}^{(2)} = \mu_{21}^{(2)} = 0,18$  - plate dimensions (Fig. 1)  $a = 1,2 \text{ m}$  vs  $= 3,6 \text{ m}$ ,

- the thickness of the concrete layer  $h = 18 \text{ sm}$ ,

-thickness of the fiberglass layer  $\delta_n = 0,25 \text{ sm}$ ,

-seam thickness  $h_{\text{III}} = 0,015 \text{ sm}$ .

Seam shear modules  $G_{\text{III}13}$  and  $G_{\text{III}23}$  they ranged from 1 before 50000.

The calculation results showed that the change (increase) in the shear modulus of the seam  $G_{\text{III}ik}$  from 60 before 600 MPa leads to a 3.9% decrease in deflection, while the load-bearing reinforced concrete layer increases by 8.1%.

From the obtained dependencies, it can be seen that the smaller the shear modulus of the seam is compared to the layer ( $G_{\text{III}ik} < G_{ik}^{(1)}, G_{\text{III}ik} < G_{ik}^{(2)}$ ), The effect of seam compliance on the stress-strain state of double-layer composite reinforced concrete slabs is greater.

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