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RANDOM VARIABLES AND THEIR NUMERICAL CHARACTERISTICS

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Abstract

The widely practiced technique of conceiving of results of measurements as realizations of random variables is investigated in this paper. Theorems are presented in which the structure of a probability space is derived from well-known representation theorems of measurement theory. These theorems are related to the theory of qualitative probability representations. Furthermore representations are shown to be random variables, if the probability space on the measurement structure satisfies a natural condition. Moreover, it is shown how independent random variables which are required by most statistical applications can be constructed in this framework.

A random variable is called such a value which takes a previously unknown numerical value as a result of experience. The values that it can take as a result of experience form a set of its possible values or a spectrum of values. Random variables may be continuous or discrete. We will denote random variable by X, and its possible values by x.

For example, let X be the number of points dropped on the die roll. X is a random variable and the set of its values will be:

$\{1,2,3,4,5,6\}$

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A random variable is called discrete, if the set of its possible values countable (i.e. all possible values can be numbered by natural numbers).

$\{X_1, X_2, \ldots\ldots, X_n\}$

A discrete random variable is completely determined by its distribution series**.** The distribution series is a table in which there are all possible values of the random variable and their probabilities:

X_i	X_1	X_2	………	X_n
P_i	P_1	P_2	………	P_n

Since the distribution series contains all possible values of the random variable, the total probability should be equal to 1. For a series of distributions, we can find various probabilities and build a distribution polygon.

Distribution polygon - polyline, which connects the points abscissas of which contains the first row of the distribution series (values of a random variable), and the ordinates are the second Distribution polygon - polyline, which connects the points abscissas of which contains the first row of the distribution series (values of a random variable), and the ordinates are the second line (the probabilities of these values).line (the probabilities of these values).

EXAMPLE:

Consider an experiment with rolling two dice. Let the random variable X be the sum of the points dropped. Let's construct a distribution series for it:

X_i	2	3	4	5	6	7	8	9	10	11	12
P_i	1/36	2/36	3/36	4/36	5/36	6/36	7/36	8/36	9/36	10/36	11/36

Let's find the probability of the following events:

$P(X<5)$, $P(X>10)$, $P(3<X<7)$.

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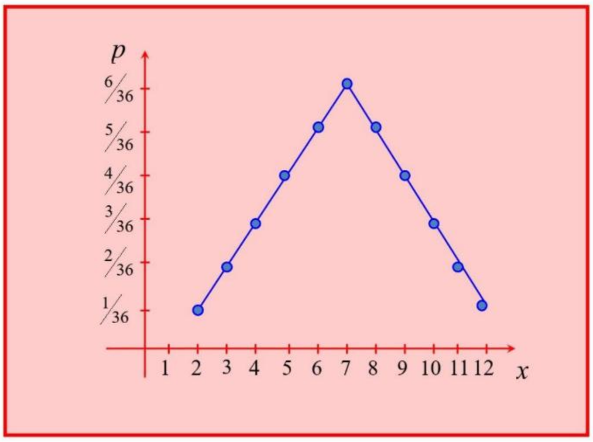
$P(X<5)=P(X=2)+P(X=3)+P(X=4)==1/36+2/36+3/36=6/36=1/6$
$P(X>10)=P(X=11)+P(X=12)==2/36+1/36=3/36=1/12$
$P(3<X<7)= P(X=4)+P(X=5)+P(X=6)==3/36+4/36+5/36=12/36=1/3$
Let's construct a distribution series:



EXPECTATION AND ITS PROPERTIES

Let X be a discrete random variable given by its distribution series:

X_1	……..	X_i	……..	X_n
P_1	…….	P_i	……..	P_n

The mathematical expectation M [X] of random variable X is called the sum:

$$M[X] = m_x = \sum_{i=1}^{n} x_i \cdot p_i$$

For example, in the above case with two dice:

$$M[X]=2*\frac{1}{36}+3*\frac{2}{36}+4*\frac{3}{36}+5*\frac{4}{36}+6*\frac{5}{36}+7*\frac{6}{36}+8*\frac{5}{36}+9*\frac{4}{36}+10*\frac{3}{36}+11\frac{2}{36}+12\frac{1}{36}=7$$

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THEOREM:

Arithmetic mean of values, taken as a random variable in a long series of experiments, is equal approximately to its mathematical expectation.

This theorem expresses an approximate relationship between the arithmetic mean and mathematical expectation. Let's carry out ***n*** experiments[1].

Let the random variable *X* take the value x_1- n_1 times, x_2 - n_2 times, ..., x_m - x_m times**.** Let's find the arithmetic mean of this random variable:

$$S = (x_1 n_1 + x_2 n_2 + \dots + x_m n_m) \cdot \frac{1}{n} \quad x_1 \cdot \frac{n_1}{n} + x_2 \cdot \frac{n_2}{n} + \dots + x_m \cdot \frac{n_m}{n}$$

Since the ratio of the form n_i / n determines the frequency of an event in a given series of experiments, then with a sufficiently large number of experiments it approaches the probability of this event:

$$x_1 p_1 + x_2 p_2 + \dots + x_m p_m = M[X]$$

PROPERTIES OF THE MATHEMATICAL EXPECTATION

1) The mathematical expectation of constant is equal to this constant:

$M[C] = C,\ C = const.$

2) The mathematical expectation of the sum of random variables *X* and *Y* is equal to the sum of mathematical expectation of of these variables:

$M[X+Y]=M[X]+M[Y]$

3) The mathematical expectation of the sum of random variable *X* and constant *C* is equal to the sum mathematical expectation of *X* and the value *C* itself:

$M[X+C]=M[X]+C$

4) A constant va lue can be carried out of a math expectation sign:

$M[k X]=k\, M[X]$*, where k=const.*

[1] Yakubova, U., Parpieva, N., & Mirhojaeva, N. (2021). Some Applications of Matrix Theory in Economics. Bulletin of Science and Practice, 7(2), 245-253. (in Russian).

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5) Expectation of a product of independent random variables X and Y is equal to the product of mathematical expectations of these variables:

$M[XY]=M[X]M[Y]$

DISPERSION AND ITS PROPERTIES

Dispersion is a measure of scattering of the values of a random variable near its mathematical expectation:

$$D[X] = M[(X - m_x)^2]$$

For example, let a random variable X be given by a distribution series:

X	0	1
P	q	p

Let's find the mathematical expectation and dispersion of this random variable.

$$M[X] = 0 \cdot q + 1 \cdot p = p = m_x$$

$$D[X] = (0 - m_x)^2 \cdot q + (1 - m_x)^2 \cdot p =$$

$$= (0 - p)^2 \cdot q + (1 - p)^2 \cdot pp^2 \cdot q + q^2 \cdot p =$$

$$= p \cdot (1 - p) = pq$$

To calculate the dispersion, another formula is often used:

$$D[X] = M[X^2] - m_x^2$$

DISPERSION PROPERTIES

1) Dispersion from a constant is equal to zero:

$D[C]=0, \quad C=const$

2) The dispersion of the sum of the random variable X and a constant C is equal to the dispersion of X:

$D[X+C]=D[X]$

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3) Constant may be carried out of the dispersion sign squared:
$D[kX]=k^2 D[X]$
4) The dispersion is always non-negative:
D[x]>=0
5) Dispersion of the sum of two random variables is found by the formula:
$$D[X+Y]=D[X]+D[Y]+2K_{XY}$$
6) The quantity K_{XY} is called correlation moment of random variables X and Y:
$$K_{XY}=M[(X-m_x)(Y-m_y)]$$
The correlation moment describes the interaction of two random variables. If the random variables *X* and *Y* are independent, then their correlation moment is 0. The square root of the dispersion is called the standard deviation:
$$\sigma[X]=\sqrt{D[X]}=\sigma_x$$
The dispersion has the dimension of the square of the random variable, and standard deviation has the dimension of the random variable itself [3].

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